THE STUDY OF OPTIMAL CONTROL PROBLEMS IN THE DISTRIBUTED COMPUTING ENVIRONMENT

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Abstract*: One of the most effective methods for solving optimal control problems with mixed constraints is a continuation method for optimal trajectories. This method, unlike the direct methods, allows you to track the qualitative features of the behavior of optimal trajectories, which is especially important when solving applied tasks.*

KEYWORDS*: OPTIMAL CONTROL, DISTRIBUTED COMPUTING.*

1. Introduction

One of the most effective methods for solving optimal control problems with control-state constraints is a continuation method for optimal trajectories [1], [2]. This method, unlike the direct methods, allows you to track the qualitative features of the behavior of optimal trajectories, which is especially important when solving applied tasks. As a result of application of the method of continuation of optimal trajectories there is a sequence of modes. Each mode is defined by solving the special problems of mathematical programming. And the mode is the Cauchy problem for systems of ordinary differential equations. A significant feature of the proposed approach is the need to control the dependence of solutions from initial conditions. Therefore, it is advantageous to apply the methods of solving Cauchy problems for systems of ordinary differential equations based on an approximate analytic form of the solutions [3].

The proposed approach allows a natural decomposition of the optimal control problem and bring the solution to the model subproblems: the solution of finite-dimensional mathematical programming problems to systems for determining shift points; computing mode as a result of solving problems of mathematical programming. It is therefore advisable to carry out the numerical solution of this problem in a distributed computing environment. Typical subtasks are placed in the nodes of the computer network and issued in the form of web services. Management scenarios in a distributed environment MathCloud [4], [5], [6] is proposed for the implementation of the algorithm.

2. *The optimal control problems with control-state constraints*

The optimal control problems with control-state constraints can be written in the form,

$$J[u]=\int_0^T F_0(x(t),\,u(t))\,d\,t\to \min,$$
$$\dot{x}(t)=u(t),\quad x(0)=x_0$$
$$G(x(t),\,u(t))\le 0,$$
$$F(x(t),u(t))=0,$$

where $x(\cdot):[0,T]\to R^n,\ F(\cdot):R^n\times R^n\to R,\ F(\cdot):R^n\times R^n\to R^m,$

$$m<n,\ G(\cdot):R^n\times R^n\to R^k.$$

In this paper we will study the optimal control problems with control-state constraints
and linear controls. The problem (Problem A) can be written in the form

$$\int_0^T \langle g(x(t)),u(t)\rangle dt\to \min$$
$$\dot{x}(t)=u(t),\ \ x(t_0)=x_0,$$
$$K(x(t))\cdot u(t)=L(x(t)),$$
$$M(x(t))\cdot u(t)\ge N(x(t)),$$
$$x(t)=(x_1(t),...,x_n(t)),\ u=(u_1(t),...,u_n(t))$$
$$K(x(t))\ -k\times n\ matrix,\ M(x(t))\ -m\times n\ matrix$$
$$L(x(t))\ -matrix\ k\times 1,\ N(x(t))\ -matrix\ m\times 1$$

3. *Local optimal control problem with linear controls*

Let us connect the linear programming problem (Problem B)

$$\langle g(x_0),u\rangle\to \min$$
$$K(x_0)\cdot u=L(x_0),$$
$$M(x_0)\cdot u\ge N(x_0),u=(u_1,...,u_n),\ x_0=(x_{01},...,x_{0n})$$
$$K(x_0)\ -k\times n\ matrix,\ M(x_0)\ -m\times n\ matrix$$
$$L(x_0)\ -matrix\ k\times 1,\ N(x_0)\ -matrix\ m\times 1$$

with Problem A. And let the Problem B has got a regular solution. This means

$$\begin{array}{ll}\langle g(x_0),u^*\rangle=\min, & \int_0^T\langle g(x(t)),u(t)\rangle dt=\min,\\ K(x_0)\cdot u^*=L(x_0), & \dot{x}(t)=u(t),\ \ x(t_0)=x_0,\\ M_A(x_0)\cdot u^*=N_A(x_0), & K(x(t))\cdot u(t)=L(x(t)),\\ M_P(x_0)\cdot u^*>N_P(x_0), & M_A(x(t))\cdot u(t)=N_A(x(t)),\\ & M_P(x(t))\cdot u(t)>N_P(x(t)),\end{array}$$

where A – indexes of active constrains and P - indexes of passive constrains.

Theorem There exist $[0,T]$, that if $t\in[0,T]$, and then

$$u(t)=\dot{x}(t)=\begin{pmatrix}K(x(t))\\ M_A(x(t))\end{pmatrix}^{-1}\begin{pmatrix}L(x(t))\\ N_A x(t))\end{pmatrix},\ x(0)=x_0,\ \ Caucshy\ \ problem.$$

4. *Continuation of the optimal trajectories in the optimal control problems with control-state constraintand linear controls*

Local optimal control problems give us an ability to restore the optimal trajectory for Problem A.

Let $x^*(t)$ - is the optimal trajectory of the Problem A.

Then continuation of the optimal trajectory to $[0, T+\Delta]$ is defined by the problem

$$\int_t^{t+\Delta} \langle g(x(\tau)) + \mu(\tau), u(\tau) \rangle d\tau \to \min, \quad t \in [0, T],$$
$$\dot{x}(\tau) = u(\tau), \quad x(t) = x^*(t),$$
$$K(x(\tau)) \cdot u(\tau) = L(x(\tau)),$$
$$M(x(\tau)) \cdot u(\tau) \geq N(x(\tau)).$$

5. *Computing of the optimal trajectories in the optimal control problems with control-state constraints and linear controls in distributed environment*

Required resources to solve the optimal control problems with mixed constraints and linear controls in distributed environment

- Finding roots of algebraic equations
  - Solution of problems of mathematical programming (LP, EP, BP)
  - Finding the dependence of the solution of the ODE system from the initial conditions, which allows several ways of distributed solutions, for example, the principles represent a solution in the form of formal "character" series
  - Resources that produce symbolic computation
  Calculating the values of polynomials in many variables (generalized Horner scheme).

The flowchart of the algorithm is shown below.

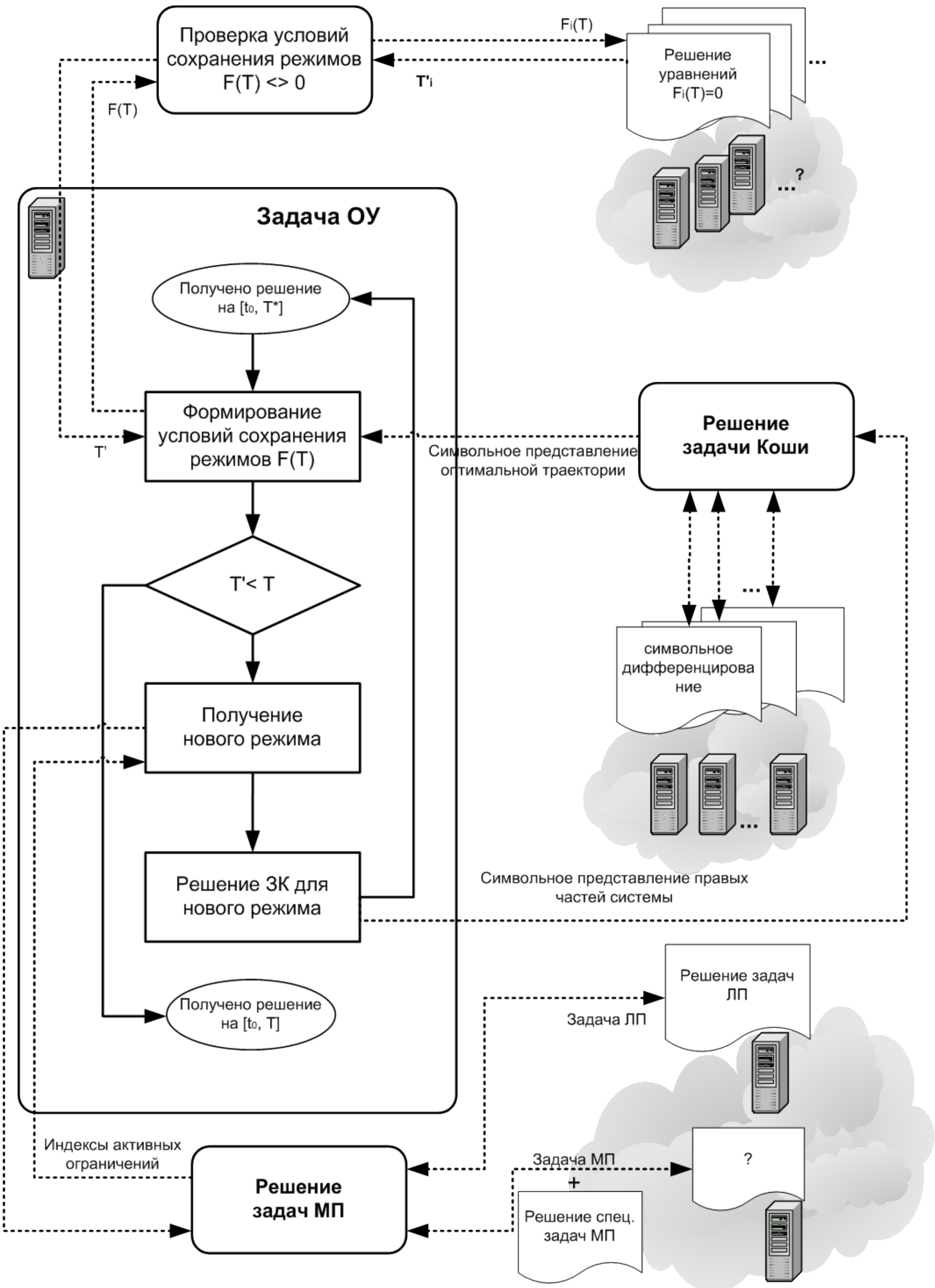

Figure 1. Flowchart of the algorithm.

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